



## DESIGN OF DIGITAL FIR FILTER USING GREY WOLF OPTIMIZER ALGORITHM

Navdeep Kaur Sidhu And J.S. Dhillon

Electrical and Instrumentation Engineering Department

Sant Longowal Institute of Engineering and Technology Longowal, Sangrur

Punjab, India

**Abstract:** In this paper, Grey Wolf Optimizer is applied to design digital FIR filters. The GWO algorithm mimics the hunting mechanism and leadership hierarchy of grey wolves. In GWO algorithm, four types of wolves such as alpha, beta, delta and omega are engaged for simulating the leadership hierarchy. The design of digital FIR filters involves the computation of best optimal filter coefficients which trying to meet ideal filter characteristics. Three key parameters which are responsible for filter performance are maximum pass-band ripple, maximum stop-band ripples and stop-band attenuation. The results of proposed GWO based approach has been compared with other optimization methods available in literature. Results reveal that the FIR filter design approach by using GWO outperforms other techniques undertaken for comparison in terms of pass-band ripples, stop-band ripples and stop-band attenuation.

**Keywords:** Real Coded Genetic algorithm (RCGA), Particle Swarm Optimization (PSO), Differential Evolution (DE), Cat Swarm Optimization (CSO), Gravitational Search Algorithm (GSA), Cuckoo Search Algorithm (CSA) and Grey Wolf Optimizer (GWO), Finite Impulse Response (FIR) and Infinite Impulse Response (IIR).

### I. INTRODUCTION

Filtering is the most extensively used process in signal processing. Digital filters play critical role in digital signal processing applications such as image processing and communication system etc. The applications of digital filters are spreading widely. In communication system, digital filters play important role in eliminating the interference. In image processing, digital filters are selected to improve the quality of distorted images. Digital filters are applied in the processing of biomedical signals such as EEG, ECG and MRI images. Filters allow the reforming the spectrum of input signal to obtain desired spectral characteristic in the output signal. There are two types of digital filters i.e. Infinite Impulse Response (IIR) and Finite Impulse Response (FIR) [1]. IIR filters have some merits such as less filter coefficients and requires less memory. But the disadvantages of IIR filter is nonlinear phase response and less stability. In comparison, FIR filters have characteristics such as linear phase and stability. Due to these advantages FIR filters are more preferred than IIR filters [2].

There are many classical methods available for the designing of digital FIR filters such as window method, frequency sampling method etc. Various types of windows such as Kaiser, Hamming, Hanning and Blackman are used for the efficient design of filter which depends upon the specification of filter [3,4]. With the help of windows function, the infinite impulse response of filter is approximated to finite impulse response. The maximum ripples amplitude in filter response is fixed regardless of order of filter. But disadvantage of these methods is that control on the pass-band edge frequencies, stop-band edge frequencies and transition band is not proper. In the optimal method, for filter design the objective is to determine the filter coefficients, such that the maximum error is minimized in the pass-band and stop-band regions [3]. The filter coefficients is searched by using optimization techniques.

Classical optimization methods cannot minimize the objective function which are non-differentiable and multimodal. To avoid the drawbacks of these methods, researchers have utilized heuristic evolutionary optimization algorithm based on natural selection and evolution techniques. Gravitational Search Algorithm (GSA) [5], Cat Swarm Optimization (CSO) [6], Cuckoo Search Algorithm (CSA) [7], Particle Swarm Optimization (PSO) [8-14], Real Coded Genetic algorithm (RCGA) [15], Genetic Algorithm (GA) [16-17], Differential Evolution (DE) [19-21] and Artificial Bee Colony (ABC) [22-24] were used for the design of digital FIR filters.

Most of the above algorithms have some problems such as stagnation to local solution, premature convergence and revisiting of same solution again and again. Some of these algorithms minimizes the ripples in the pass-band but not minimizes the ripples in stop-band [6]. In order to overcome these problems, Grey Wolf Optimizer algorithm method has been proposed to design FIR filter. GWO is generated by observing the hunting behavior of grey wolves. In GWO, four types of wolves alpha, beta, delta and omega are involved. The position of prey is estimated by alpha, beta and delta and other wolves update their position randomly around the prey [27]. GWO keeps the best solution until it reaches the end of iteration. The final solution is the best position of alpha search agent. In this paper, the optimal designs of FIR low-pass (LP) and high-pass (HP) filters have been discussed by using GWO algorithm. In order to, prove the merits of this algorithm comparison is performed with RCGA, PSO, DE and CSO.

### II. PROBLEM FORMULATION

In this section problem formulation for FIR filter design is described. The Z transform of FIR filter is mathematically defined as:

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n} \quad (1)$$

Where  $h(n)$  is impulse response of FIR filter,  $N-1$  is order of filter and  $N$  is length of filter. The value of  $h(n)$  determines the type of filter i.e., Low-pass (LP), High-pass (HP), Band-pass (BP), Band-stop (BS) etc. While designing the filter, low distortion, sharp transition band and highest stop-band attenuation are some specifications. There are some ways of formulating the designed problem presented in literature survey. In this paper, Type 1 linear phase filter with odd length and symmetric coefficients is considered. The length of  $h(n)$  is  $N$ , due to symmetry property  $h(n) = h(N-n-1)$ . So, the value of  $M = (N-1)/2$ .

$$H_d(\omega) = h(M) + 2 \sum_{n=0}^{M-1} h(n) \cos((M-n)\omega) \quad (2)$$

where  $H_d(\omega)$  represents magnitude response of designed filter.

One of the major advantages of designing the linear phase FIR filter is that its coefficients are symmetric. Only half of the coefficients are calculated then they are concatenated to obtain total  $h(n)$  sequence. Due to this symmetry of coefficients, dimensions of the problem is halved and computational burden is also reduced [2,5,16].  $H_i(\omega)$  represents the magnitude response of the ideal filter for low-pass and high-pass filter as shown in (3) and (4).

$$H_i(\omega) = \begin{cases} 1 & ; \text{for } 0 \leq \omega \leq \omega_c \\ 0 & ; \text{otherwise} \end{cases} \quad (3)$$

$$H_i(\omega) = \begin{cases} 0 & ; \text{for } 0 \leq \omega \leq \omega_c \\ 1 & ; \text{otherwise} \end{cases} \quad (4)$$

where  $\omega_c$  is the cut off frequency for low pass and high pass filter. The objective function is to determine the filter coefficients such that value of error is minimized. The error is the difference between the magnitude response of designed filter and the magnitude response of ideal filter. In this paper, error fitness function has been adopted. In this function, summation of absolute error for whole frequency band is considered, but in other functions only maximum error is considered. Using (5), it has been found that the ripples in pass-band and stop-band are minimized.

$$f(\omega) = \sum_{\omega=0}^{\omega_c} \left| |H_d(\omega)| - H_i(\omega) \right| - \delta_p + \sum_{\omega_c+1}^{\pi} \left| |H_d(\omega)| - H_i(\omega) \right| - \delta_s \quad (5)$$

where  $\delta_p$  and  $\delta_s$  are ripples in passband and stopband. The error function given in (5) represents the fitness function to be minimized by using RCGA, PSO, DE, CSO and GWO.

### III. GREY WOLF OPTIMIZER ALGORITHM

GWO is inspired by the hunting behavior of grey wolves. Four types of grey wolves are considered based on their natural activity such as alpha, beta, delta and omega. The leaders are male and female, called alphas. The decision about hunting, sleeping and place to walk is made by alpha. The decision of alphas should be tracked by other members of pack. The alpha is also responsible for supervision of the pack. The betas are wolves are responsible for helping the alphas in decision making. The beta wolves can be male or

female. In case alpha's passes away then beta is considered as best candidate. The beta wolves commands the other low level wolves, but always respect alpha wolves. It plays the role of an advisor to the alpha and discipliner for wolves. The beta gives feedback to alpha and reinforces the alpha's commands throughout the wolves. The lowest ranking grey wolf is omega. Delta wolves have to submit to alphas and betas. The main phases of grey wolf hunting are as:

- Tracking, chasing and approaching the prey.
- Pursuing, encircling and harassing the prey until it stops moving
- Attack towards the prey.

Alpha, beta and delta estimate the position of prey then other wolves update their position according to the positions of these three best search agents [27].

Steps of GWO are as follows:

Step 1: Initialization: filter order =  $Z$ ; population of search agents,  $N_p$ ; number of variables = number of filter coefficient,  $N_v = (Z/2)+1$ , minimum and maximum value of filter coefficients  $h_{min} = -1, h_{max} = 1$

Step 2: Generate the initial vector of filter coefficients,  $(Z/2)+1$  randomly with limits by using (6).

$$h_{ij} = h_{min} + rand(0,1)(h_{max} - h_{min}) \quad (i = 1, 2, \dots, N_p; j = 1, 2, \dots, N_v) \quad (6)$$

Compute the fitness of total population  $N_p$  by using (5).

Step 3: Arrange the function values in ascending order and computational of three best solution vectors  $h_{1j}, h_{2j}$  and  $h_{3j}$  corresponds to  $f_1, f_2$  and  $f_3$ .

Step 4: Updating the position of other wolves by using equations given below. The A and C are coefficient vectors. The vector A and C are calculated as follows:

$$A_i = a(2r_i - 1) \quad (i = 1, 2, \dots, 3) \quad (7)$$

$$C_i = 2r_{j+i} \quad (i = 1, 2, \dots, 3; j = 3 + i) \quad (8)$$

where  $r_1, r_2, r_3, r_4, r_5$  and  $r_6$  are random vector in  $(0,1)$ .  $a$  is decreased from 2 to 0 over the course of iterations.

$$D_{kj}^t = |C_k h_{kj}^t - h_{ij}^t| \quad (k = 1, 2, \dots, 3; j = 1, 2, \dots, N_v; i = 1, 2, \dots, N_p) \quad (9)$$

$$x_{kj}^t = h_{kj}^t - A_k D_{kj}^t \quad (k = 1, 2, \dots, 3; j = 1, 2, \dots, N_v; i = 1, 2, \dots, N_p) \quad (10)$$

$$h_{ij}^{t+1} = \frac{1}{3} \sum_{k=1}^3 x_{kj}^t \quad (j = 1, 2, \dots, N_v; i = 1, 2, \dots, N_p) \quad (11)$$

checking the limits of filter coefficients and evaluate the fitness of total population  $N_p$ .

Step 5: Update the  $h_{1j}, h_{2j}$  and  $h_{3j}$  on basis of updated fitness values.

Step 6: Iterations continues from step 4 till the maximum number of iterations are reached.

Step 7:  $h_{1j}$  is the vector of optimal filter coefficients  $(Z/2)+1$ . By copying and concatenation of these optimal coefficients form complete filter coefficients.

### IV. SIMULATION RESULTS

Results are reported after giving 50 runs. Each filter of order 20 and number of coefficients or length of filter 21. Number of frequency samples = 128, number of population = 100 and maximum iteration cycles = 500. The parameters

of filter to be designed by using GWO are pass-band ripples  $\delta_p$  is set to 0.1, and stop-band ripples  $\delta_s$  is set to 0.01. For the lowpass filter, pass-band edge frequency is  $0.45\pi$  and stopband edge frequency is  $0.55\pi$ . For the high-pass filter, stop-band edge frequency is  $0.45\pi$  and passband edge frequency is  $0.55\pi$ .

Table 1-2 show the optimized coefficients of lowpass and highpass FIR filter. Table 3-4 summarize the mean, variance and standard deviation of pass-band ripples and stop-band ripples for lowpass filter using RCGA, PSO, DE, CSO and

GWO. Figure 1 and Figure 3 show magnitude response for low-pass filter and high-pass filter. Figure 2 and Figure 4 show magnitude response in db for low-pass filter and high-pass filter. Table 5 and Table 8 show the maximum stop-band attenuation of low-pass and high-pass filter of order 20 by RCGA, PSO, DE, CSO and GWO. Table 6-7 summarize the mean, variance and standard deviation of pass-band ripples and stop-band ripples for high-pass filter using RCGA, PSO, DE, CSO and GWO.

Table 1. Optimized coefficients of the LP filter of order 20

Optimized coefficients	RCGA	PSO	DE	CSO	GWO
h(1)=h(21)	0.0206445	0.0251167	0.0270053	0.0288986	0.0141067
h(2)=h(20)	0.0487214	0.0472192	0.0472668	0.0474313	0.0274373
h(3)=h(19)	0.0058686	0.0035462	0.0053202	0.0058656	-0.0054660
h(4)=h(18)	-0.0409668	-0.0400940	-0.0389822	-0.0357453	-0.0342177
h(5)=h(17)	-0.0008635	-0.0005204	-0.0034522	0.0013515	0.0054833
h(6)=h(16)	0.0597960	0.0609072	0.0579468	0.0596294	0.0556549
h(7)=h(15)	-0.0014088	-0.0017592	-0.0020514	0.0039075	-0.0063680
h(8)=h(14)	-0.1031178	-0.1036139	-0.1027152	-0.1031154	-0.1013287
h(9)=h(13)	-0.0004406	0.0006276	0.0016929	0.0029165	0.0065438
h(10)=h(12)	0.3176006	0.3181190	0.3197956	0.3186922	0.3165144
h(11)	0.5000185	0.5000185	0.5000185	0.5048214	0.4932865

Table 2. Optimized coefficients of the HP filter of order 20

Optimized coefficients	RCGA	PSO	DE	CSO	GWO
h(1)=h(21)	0.0217313	0.0255591	0.0290419	0.0275382	0.01350132
h(2)=h(20)	-0.0481316	-0.0474136	-0.0458732	-0.0443804	-0.02770845
h(3)=h(19)	0.0062981	0.0051354	0.0029505	0.0032993	-0.00556139
h(4)=h(18)	0.0418953	0.0399880	0.0413117	0.0429334	0.03442397
h(5)=h(17)	0.0008799	0.0014059	-0.0002839	0.0000993	0.00507274
h(6)=h(16)	-0.0590278	-0.0602831	-0.0600023	-0.0582345	-0.05549605
h(7)=h(15)	-0.0000135	0.0007686	-0.0039211	0.0031813	-0.00648871
h(8)=h(14)	0.1042576	0.1051207	0.1061191	0.1024026	0.10165160
h(9)=h(13)	0.0038237	0.0014719	-0.0005650	-0.0022363	0.00606649
h(10)=h(12)	-0.3166314	-0.3154715	-0.3200839	-0.3179255	-0.3167072
h(11)	0.4994680	0.4999814	0.4999814	0.4999814	0.4939843

Table 3. Quantitative analysis of passband ripple for FIR lowpass filter of order 20

Algorithm	Maximum value of ripples	Minimum value of ripples	Mean value of ripples	Variance of ripples	Standard deviation of ripples
RCGA [5]	0.114	0.112	0.113	0.000001	0.001
PSO [5]	0.123	0.116	0.1193	0.000008	0.00287
DE [5]	0.135	0.113	0.124	0.000121	0.011
CSO [5]	0.164	0.122	0.1363	0.0003829	0.01957
GWO	0.0703	0.0708	0.0705	0.00000049	0.0007

Table 4. Quantitative analysis of stopband ripple for FIR lowpass filter of order 20

Algorithm	Maximum value of ripples	Minimum value of ripples	Mean value of ripples	Variance of ripples	Standard deviation of ripples
RCGA [5]	0.04949	0.01196	0.025522	0.000197	0.014026
PSO [5]	0.03967	0.01089	0.018986	0.000121	0.010983
DE [5]	0.03339	0.009651	0.01609	0.0000751	0.008666
CSO [5]	0.01998	0.012222	0.01655	0.0000079	0.002811
GWO	0.0297	0.0002	0.0150	0.000001	0.0010

Table 5. Comparison of stopband attenuation of lowpass FIR of order 20 using different algorithms

Algorithm	RCGA [5]	PSO [5]	DE [5]	CSO [5]	GWO
Stop-band Attenuation	26.11	28.03	29.53	33.99	30.5

Table 6. Quantitative analysis of passband ripple for FIR highpass filter of order 20

Algorithm	Maximum value of ripples	Minimum value of ripples	Mean of ripples	Variance of ripples	Standard deviation of ripples
RCGA [5]	0.117	0.109	0.113	0.000016	0.004
PSO [5]	0.122	0.111	0.118	0.00002467	0.004966
DE [5]	0.136	0.108	0.125	0.0001487	0.01219
CSO [5]	0.132	0.111	0.118	0.000098	0.009899
GWO	0.0669	0.0726	0.0697	0.00000049	0.0007

Table 7. Quantitative analysis of stopband ripple for FIR highpass filter of order 20

Algorithm	Maximum value of ripples	Minimum value of ripples	Mean of ripples	Variance of ripples	Standard deviation of ripples
RCGA [5]	0.05461	0.01405	0.028603	0.000254	0.015925
PSO [5]	0.03935	0.01125	0.01916	0.000111	0.010525
DE [5]	0.03483	0.008113	0.016107	0.0000919	0.009585
CSO [5]	0.02085	0.01258	0.01611	0.00001	0.003245
GWO	0.0224	0.0002	0.0113	0.00000441	0.0021

Table 8. Comparison of stopband attenuation of highpass FIR of order 20 using different algorithms

Algorithm	RCGA [5]	PSO [5]	DE [5]	CSO [5]	GWO
Stop-band Attenuation	25.25	28.1	29.16	33.62	32.99

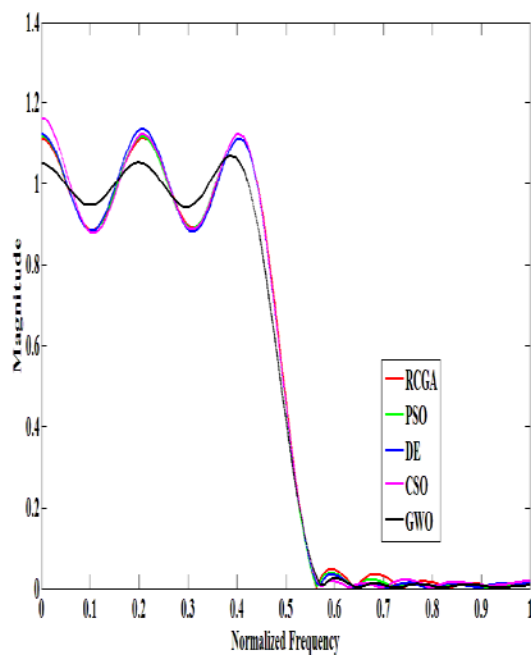


Figure 1. Magnitude response for lowpass FIR filter of order 20

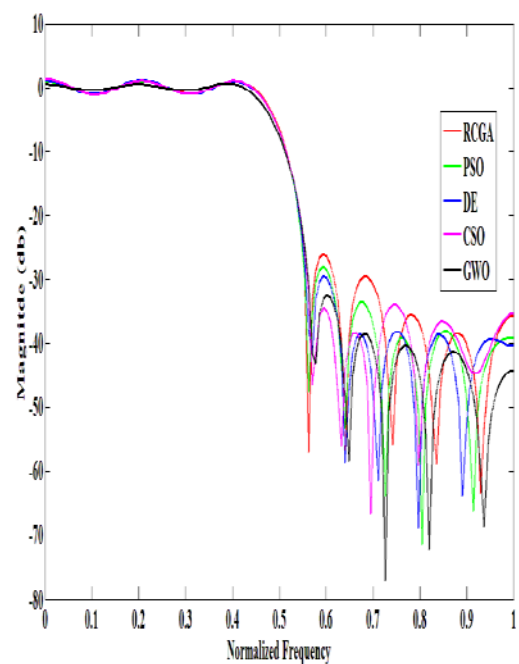


Figure 2. Magnitude response (db) for lowpass FIR filter of order 20

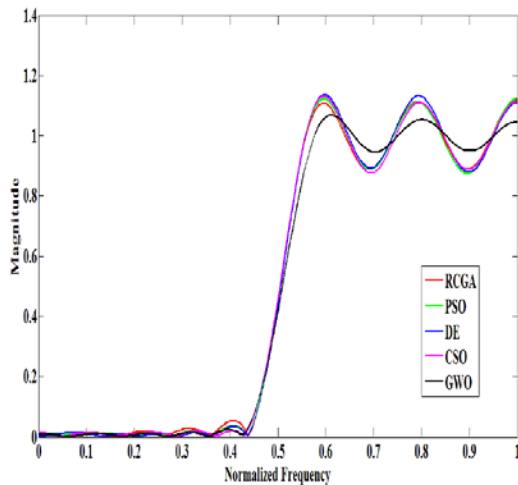


Figure 3. Magnitude response for highpass FIR filter of order 20

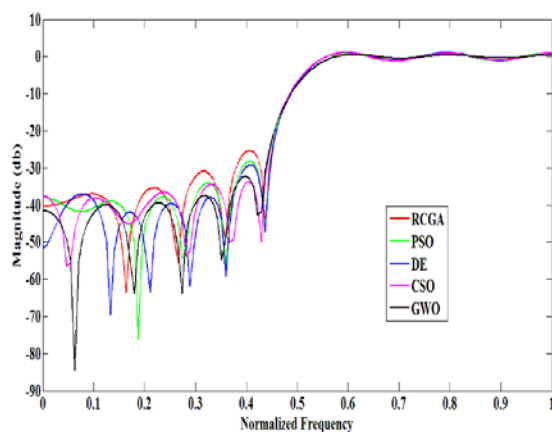


Figure 3. Magnitude response (db) for highpass FIR filter of order 20

## V. CONCLUSION

The aim for the designing of digital filter is to determine such filter coefficients that minimize the absolute error with desired filter response. In this paper, GWO algorithm is applied for the designing of digital FIR low-pass and high-pass filters. Comparison of results of RCGA, PSO, DE, CSO and GWO algorithms have been done. The simulation results indicate that the GWO reveals best performance in terms of pass-band ripples, stop-band ripples and maximum stop-band attenuation. The superiority of GWO algorithm is that its functionality depends upon few parameters adjustment which increases its convergence rate. The optimal solution is obtained with a good balance between exploitation and exploration by GWO algorithm. From the obtained results, it can be said that the proposed algorithm can be used in future for the designing of digital filters.

## VI. REFERENCES

- [1]. Proakis J.G. and Manolakis D.G. (1996), "Digital Signal Processing, Principle, Algorithm and Applications", Prentice Hall, Inc, New Jersey.
- [2]. Shao P., Wu Z., Zhou X. and Tran D.C. (2015), "FIR digital filter design using improved particle swarm optimization based on refraction principle", *SoftComputing*, vol. 21(10), pp. 2631-2642.
- [3]. Ifeachor E.C., Jervis B.W. (2002), "Digital signal processing", A Practical Approach, Pearson Education.
- [4]. Barapate R.A. and Katre J.S. (2007), "Digital Signal Processing", Tech-max publication, Pune.
- [5]. Saha S.K., Kar R. Mandal D., and Ghoshal S.P. (2013) "Design and simulation of FIR band pass and band stop filters using gravitational search algorithm", *Journal of Memetic Computing*, vol. 5(4), pp. 311-321.
- [6]. Saha S.K., Ghoshal S.P., Kar R. and Mandal D. (2013), "Cat swarm optimization algorithm for optimal linear phase FIR filter design", *ISA transactions*, vol. 52(6), pp.781-794.
- [7]. Kwan H.K. and Liang J. (2016), "Minimax design of linear phase FIR filters using cuckoo search algorithm", 2016 8th International Conference on Wireless Communications and Signal Processing, Yangzhou, China, 13-15 Oct., 2016, pp. 1-4.
- [8]. Mandal S., Ghoshal S.P., Kar R., and Mandal D. (2012), "Design of optimal linear phase FIR high pass filter using craziness based particle swarm optimization technique", *Journal of King Saud University-Computer and Information Sciences*, vol. 24(1), pp. 83-92.
- [9]. Ababneh J.I. and Bataineh M.H. (2008), "Linear phase FIR filter design using particle swarm optimization and genetic algorithms", *Digital Signal Processing*, vol. 18(4), pp. 657-668.
- [10]. Chang W.D., and Chang D.M. (2008), "Design of a higher-order digital differentiator using a particle swarm optimization approach", *Journal of Mechanical Systems and Signal Processing*, vol. 22(1), pp. 233-247.
- [11]. Aggarwal A., Rawat T.K. and Upadhyay D.K. (2016), "Design of optimal digital FIR filters using evolutionary and swarm optimization techniques", *International Journal of Electronics and Communications*, vol. 70(4), pp. 373-385.
- [12]. Najjarzadeh M. and Ayatollahi A. (2008a), "FIR digital filters design: particle swarm optimization utilizing LMS and minimax strategies", *IEEE International Symposium on Signal Processing and Information Technology*, 16-19 Dec., 2008, pp. 129-132.
- [13]. Huang W.P., Zhou L.F. and Qian J.X. (2004), "FIR filter design: frequency sampling filters by particle swarm optimization algorithm", *Proceeding of third International Conference on Machine Learning and Cybernetics*, Shanghai, China, 26-29 Aug., 2004, vol. 4, pp. 2322-2327.
- [14]. Mondal S., Chakraborty D., Kar R., Mandal D., and Ghoshal S.P. (2012), "Novel particle swarm optimization for high pass FIR filter design", *IEEE Symposium on Humanities, Science and Engineering Research*, Kuala Lumpur, Malaysia, pp. 413-418, 24-27 June, 2012.
- [15]. Maharastra, India Aggarwal A., Rawat T.K., Kumar M. and Upadhyay D.K. (2015), "Optimal design of FIR high pass filter based on L1 error approximation using real coded genetic algorithm", *International Journal of Engineering Science and Technology*, vol. 18(4), pp. 594-602.
- [16]. Rana K.P.S., Kumar V. and Nair S.S. (2016), "Efficient FIR filter designs using constrained genetic algorithms based optimization", 2nd International Conference on Communication Control and Intelligent Systems, Mathura, India, 18-20 Nov., 2016, pp. 131-135 Babu R. (2012), "Digital Signal Processing", Scitech publication, Chennai.
- [17]. Ahmad S.U. and Antoniou A. (2006), "A genetic algorithm approach for fractional delay FIR filters",

- Proceeding of IEEE International Symposium on Circuits and Systems, Island of Kos, Greece, 21-24 May, 2006, pp. 2517-2520,.
- [18]. Deb K. (2005), "Optimization for engineering design", Prentice Hall of India, New Delhi.
- [19]. Liu G., Li Y. and He G. (2010), "Design of digital FIR filters using differential evolution algorithm based on reserved genes", IEEE Congress on Evolutionary computation Barcelona, Spain, Spain, 18-23 July, 2010 pp. 1-7.
- [20]. Mandal D., Kar R., and Ghoshal S.P. (2014), "Digital FIR filter design using fitness based hybrid adaptive differential evolution with particle swarm optimization", Natural Computing, vol. 13(1), pp. 55-64.
- [21]. Karaboga N. and Cetinkaya B. (2006), "Design of digital FIR filters using differential evolution algorithm", Journal of Circuits, Systems, and Signal Processing, vol. 25(5), pp. 649-660.
- [22]. Kwan H.K. and Raju R. (2016), "Minimax design of linear phase FIR differentiators using artificial bee colony algorithm", International Conference on Wireless Communications & Signal Processing, Yangzhou, China, pp. 1-4, 13-15 Oct., 2016.
- [23]. Dwivedi A.K., Ghosh S. and Londhe N.D. (2016), "Low power FIR filter design using modified multi-objective artificial bee colony algorithm", Journal of Engineering applications of Artificial Intelligence, vol. 55, pp. 58-69.
- [24]. Ji D. Aug. (2016), "The application of artificial bee colony (ABC) algorithm in FIR filter design", International Conference on Natural Computation, Fuzzy Systems and Knowledge Discovery, Changsha, China, pp. 663-667.
- [25]. Mitra S. and Kaiser J. (1993), "Handbook for Digital Signal Processing", Wiley, New York.
- [26]. Mitra S.K. (2006), "Digital Signal Processing", Tata Mc Graw Hill edition, New Delhi.
- [27]. Mirjalili S., Mirjalili S.M. and Lewis A. (2014), "Grey wolf optimizer", Advances in Engineering Software, vol. 69, pp. 46-61.