

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

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UNSTEADY QUADRATIC CONVECTIVE FLOW OF A ROTATING NON-NEWTONIAN FLUID OVER A ROTATING CONE IN A POROUS MEDIUM

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Abstract: A mathematical model is developed to study on unsteady double diffusive flow of rotating Casson fluid from a rotating vertical cone in a porous medium. Flow in porous medium is described by Darcy's law. Quadratic variation of density with temperature and concentration is assumed in momentum equation. The equations (PDE) of the model are converted into ODE using suitable similarity transformations. A numerical method namely Runge-Kutta based shooting method is used to find the solution of resultant equations. Computational results are reported graphically on velocity, temperature and concentration fields for different values of non-linear temperature and concentration parameters, Casson fluid parameter, unsteady parameter. Increasing non-linear temperature and concentration parameter tends enhance tangential velocity and reduce azimuthal and normal velocities as well as temperature and concentration fields. This study finds applications in industries like pharmaceutical industries, aerospace technology and polymer production etc.

Keywords: Unsteady, Quadratic convection, Casson fluid, Rotating cone, Porous medium, Numerical method

I. INTRODUCTION

The analysis of rotating fluids has extended much interest now days since it has been encountered in many notable problems such as cosmic and geophysical flows. The influence of Coriolis force eases us to grasp the phenomena of earth's rotation, the behaviour of ocean transmission and galaxies formation better. Therefore, many mathematical paradigms including numerical and analytical investigations are depicted to study the impression of Coriolis force on the fluid flows. Materials that do not obey the Newtonian law of viscosity are non-Newtonian fluids such as apple sauce, drilling muds, certain oils, ketchup and colloidal and interruption solution. The study of non-Newtonian fluids has gained interest because of their massive industrial and technological devotions. Nevertheless, the Navier Stokes equations are no longer defensible to specifically describe the rheological behaviour of all non-Newtonian fluids. In view of their disputes with Newtonian fluids, several models of non-Newtonian fluids have been insinuated. Hussain et al. [1] analyzed the instability of the boundary layer over a disk rotating in an enforced axial flow. Nadeem & Saleem [2] investigated the unsteady mixed convective MHD flow in a rotating frame over a rotating cone. Salman & Sohail [3] considered metallic and buoyancy particle effects along a vertically rotating cone on an unsteady water based fluid. Saleem & Nadeem [4] examined the Eyring-Powell fluid flow with mixed convection along a rotating cone. Sohail & Salman [5] researched the analytical study of nanoparticles of a third grade fluid over a rotating vertical cone. Nadeem & Saleem [6] surveyed about rotating Jeffrey nanofluid on a rotating vertical cone with mixed convection, an optimal study. Salman et al. [7] reviewed heat generation and chemical reaction of a time dependent second order viscoelastic fluid flow on a rotating cone. Bilal et al. [8] analyzed Dufour and Soret effects with chemical reaction on dissipative slip flow along heat and mass transfer over a vertically rotating cone. Malik et al. [9] investigated variable viscosity and thermal conductivity of mixed convection dissipative viscous fluid flow over a rotating cone. Mallikarjuna et al. [10] considered chemical reaction effects on MHD convective heat and mass transfer flow past a rotating vertical cone embedded in a variable porosity regime.

With the raising attention in searching for a more efficient machinery of oil recovery, an extreme deal of interest has been lately focused on the miscible flows over a porous medium as an achievable new technology for increasing the critical oil recovery. At the present time, the industrial process of oil recovery from the dwindling oil reservoirs, used currently in field projects, is water flooding. In the cases of the inauspicious oil-water agility ratio, the competence of water flooding is reduced by the viscous fingering effect, associated with the presence of boundary variability in the oil displacement mechanism. As currently reported in the literature, the oil displacement may be substantially improved by the use of a miscible dislocating fluid. Noticeably, when the displacing fluid in a porous medium displaces the oil, miscible with the former, a mixture zone occurs, in which the concentration of the displacing fluid varies. Consequently, the size of the mixture domain during the displacement process should be predicted, in order to obtain a better performance. Chamkha et al. [11] examined the non-similar solution for natural convective boundary layer flow over a sphere embedded in a porous medium saturated with a nanofluid. Kandasamy et al. [12] researched MHD nanofluid flow over a porous vertical plate with thermal and solutal stratification. Makinde et al. [13] surveyed thermophoresis and radiative heat transfer over a convectively heated plate in a porous

medium with MHD variable viscosity reacting flow. Sheikholeslami [14] reviewed Darcy model on nanofluid flow in a porous cylinder considering the influence of Lorentz forces. Mohsen & Sadoughi [15] analyzed the mesoscopic method for MHD nanofluid flow inside a porous cavity considering various shapes of nanoparticles. Vasu et al. [16] investigated heat and mass flow past through a vertical cylinder embedded in non-Darcy porous medium with chemical reaction. Hayat et al. [17] considered three dimensional rotating flows of carbon nanotubes with Darcy-Forchheimer porous medium. Chakraborthy et al. [18] examined Ag-water nanofluid flow over an inclined porous plate embedded in a non-Darcy porous medium due to solar radiation. Ahmed et al. [19] researched MHD power law fluid flow and heat transfer analysis through Darcy Brinkman porous media in annular sector.

In view of the above applications, the present paper address heat and mass transfer of rotating Casson fluid over the rotating cone in Darcy medium. A set of coupled non-linear governing equations to the problem are solved numerically and presented graphically for the effect of flow parameters on flow characteristics (velocity, temperature and concentration fields).

II. FORMULATION OF THE PROBLEM

Consider an unsteady laminar Casson rotating fluid flow from a rotating vertical cone in a saturated porous medium. A curvilinear coordinate frame is chosen in which X, Y and Z axis are in tangential, azimuthal and normal directions respectively as shown in Fig. 1. The fluid is rotating with an angular velocity Ω_2 from the rotating cone whose angular velocity is Ω_1 . The governing boundary layer equations with Boussinesq approximation are given by:

$$X\frac{\partial U}{\partial X} + U + X\frac{\partial W}{\partial Z} = 0 \tag{1}$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + W \frac{\partial U}{\partial Z} - \frac{V^2}{X} = -\frac{V_e^2}{X} + \upsilon \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 U}{\partial Z^2} - \frac{\upsilon}{K} U +$$
(2)

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + W \frac{\partial V}{\partial Z} + \frac{UV}{X} = \upsilon \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 V}{\partial Z^2} - \frac{\upsilon}{K} V + \frac{\partial V_e}{\partial t}$$
(3)

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + W \frac{\partial T}{\partial Z} = \frac{k_e}{\rho c_n} \frac{\partial^2 T}{\partial Z^2}$$
(4)

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + W \frac{\partial C}{\partial Z} = D \frac{\partial^2 C}{\partial Z^2}$$
(5)

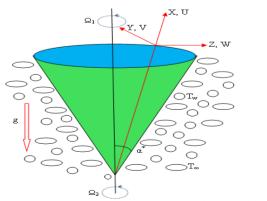


Fig1. Physical Model of the Problem

The boundary conditions are

 $U = 0, V = X\Omega_{1} \sin \alpha^{*} (1 - st^{*})^{-1}, W = 0, T = T_{w}, C = C_{w} at Z = 0 \text{ and}$ (6) $U = 0, V = V_{v}, T = T_{w}, C = C_{w} as Z \to \infty$

III. SOLUTION AND PROCEDURE

Introducing the following similarity transformations and nondimensional variables

$$\eta = \left(\frac{\Omega \sin \alpha^{*}}{\upsilon}\right)^{\prime 2} \left(1 - st^{*}\right)^{-\frac{1}{2}} Z, U = -\frac{1}{2} X \Omega \sin \alpha^{*} (1 - st^{*})^{-1} f'(\eta),$$
(7)

$$V = X \Omega \sin \alpha^{*} (1 - st^{*})^{-1} g(\eta), w = (\upsilon \Omega \sin \alpha^{*})^{\frac{1}{2}} (1 - st^{*})^{-\frac{1}{2}} f(\eta),$$
(7)

$$V_{e} = X \Omega_{2} \sin \alpha^{*} (1 - st^{*})^{-1}, \Omega = \Omega_{1} + \Omega_{2}, t^{*} = (\Omega \sin \alpha^{*}) t$$

$$T(t, X, Z) - T_{x} = (T_{w} - T_{w}) \Theta(\eta), \text{ where } T_{w} - T_{w} = (T_{0} - T_{w}) \left(\frac{X}{L}\right) (1 - st^{*})^{-2}$$

$$C(t, X, Z) - C_{w} = (C_{w} - C_{w}) \Phi(\eta), \text{ where } C_{w} - C_{w} = (C_{0} - C_{w}) \left(\frac{X}{L}\right) (1 - st^{*})^{-2}$$

The equations (2)-(6) become

$$\left(1 + \frac{1}{\beta}\right) f''' - s \left(f' + \frac{\eta}{2} f''\right) + \frac{1}{2} (f')^{2} - ff'' - Da^{-1}f' -$$
(8)

$$2g^{2} + 2(1 - \lambda)^{2} - 2\Delta \left[(\Theta + \alpha_{1}\Theta^{2}) + N \left(\Phi + \alpha_{2}\Phi^{2}\right) \right] = 0$$

$$\left(1 + \frac{1}{\beta}\right)g'' - s\left(g + \frac{\eta}{2}g'\right) - fg' + gf' - Da^{-1}g + s(1 - \lambda) = 0$$
(9)

$$\frac{1}{P_r}\Theta'' - s\left(2\Theta + \frac{\eta}{2}\Theta'\right) - f\Theta' + \frac{1}{2}f'\Theta = 0$$
(10)

$$\frac{1}{S_c}\Phi'' - s\left(2\Phi + \frac{\eta}{2}\Phi'\right) - f\Phi' + \frac{1}{2}\Phi f' = 0$$
(11)

And the associated boundary conditions are

$$f = 0, f' = 0, g = \lambda, \Theta = 1 \text{ and } \Phi = 1 \text{ at } \eta = 0 \text{ and}$$

$$f' = 0, g = 1 - \lambda, \Theta = 0 \text{ and } \Phi = 0 \text{ as } \eta \to \infty$$
 (12)

IV. RESULTS AND DISCUSSION

A set of non-linear coupled ordinary differential equations (8)-(11) subjected to the conditions (12) are solved numerically using Runge-Kutta based shooting method (Mallikarjuna et.al [20] and Srinivasacharya et.al [21]). Results shows the influence of non-dimensional governing parameters on velocity (tangential, azmithual and normal directions), temperature and concentration profiles along with the friction factor coefficients, local Nusselt and Sherwood numbers. For numerical computations we considered the non-dimensional parameter values as Da = 0.25, $\beta = 0.5$, s = 2, $\Delta = 10$, Pr = 0.71, Sc = 0.22, N = 1, $\alpha_1 = 0.5$, $\alpha_2 = 0.5$, $\lambda = 0.5$. These values are kept as common in entire study except the variations in the respective figures and tables. Table 1 guarantees the validation of the present computational solutions with already existing values presented by Hering and Grosh [22], Himasekhar and Sarma [23] and Mallikarjuna et.al[24]. We found better agreement of the results with the published literature.

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	$-\Theta'(0)$								
Δ	Hering	Himasekhar	Mallikarjuna	Present					
	and	and Sarma	et.al [24]	results					
	Grosh	[23]							
	[22]								
0	0.42852	0.4316	0.42842	0.42849					
0.1	0.46156			0.46114					
1.0	0.61202		0.61213	0.61206					
10	1.0173		1.07018	1.01728					

Table-1: Comparison results in the absence of the concentration equation for linear convection of Newtonian fluid

Figs. 2-6 display the graphical representation of velocities (tangential, azimuthal and normal directions), temperature and concentration profiles for various values of Casson fluid and unsteadiness parameters. A raise in β and s, we observed decrement and increment in the tangential ($\eta \le 3.5$) and the normal $(\eta \leq 2)$ velocity profiles, later on, increment and decrement respectively. The exactly opposite trend was seen in azimuthal velocity profiles, where as the temperature and concentration profiles are reduced. This happens due to the domination of rotation in the flow. Physically an increasing value of Casson fluid parameter improves the shear thinning properties in the flow, due to this reason we saw decrement in velocity.

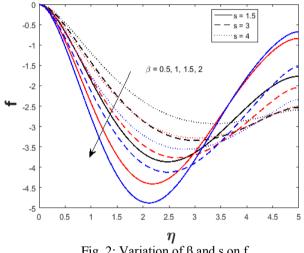
The effect of nonlinear thermal and concentration convection parameters on velocities, temperature and concentration profiles are plotted in Figs. 7-11. The improving values of α_1 and α_2 decreases the tangential velocity, temperature and concentration profiles, where as the mixed behaviour (increment and decrement) in azimuthal velocity. Similarly opposite behaviour of the azimuthal velocity profiles are observed in the normal velocity. The raising values of the nonlinear convection in the flow improve the nonlinearity between the fluid particles, this leads to reduction in temperature and concentration.

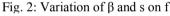
The variations of Δ and λ on velocities, temperature and concentration profiles are shown in Figs. 12-16. The improving values of α_1 and α_2 decreases the temperature and concentration profiles, the tangential and azimuthal velocity profiles are showing the mixed nature. On the other hand, except at $\lambda=0.2$ the normal velocity shows the mixed sense.

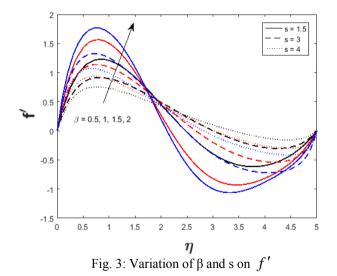
Table 2 shows the skin friction coefficients (Cf_x , Cf_y), local Nusselt (Nu) and Sherwood numbers (Sh) results for different values of α_1 , α_2 , β and s. It is observed that skin friction coefficient along tangential direction is raising with the increasing values of α_1 , α_2 and β and falling with the increasing values of unsteady parameter s, where as the skin friction coefficient along azimuthal direction is decreasing with the increasing values of α_1 and α_2 and growing with the increasing values of β and s. On the other hand the local Nusselt and Sherwood numbers are enhanced with the raise in α_1 , α_2 , and s and depreciated with increasing in β .

Table-2: Variations of friction factor coefficients, local Nusselt number and Sherwood number when Da = 0.25, $\beta =$ 0.5, s = 2, Δ = 10, Pr = 0.71, Sc = 0.22, N = 1, α_1 = 0.5, α_2 = $0.5, \lambda = 0.5$ Cfx Cfy Nu β S Sh α_1 α_2

0.5	0.5	0.5	0.2	9.5181	0.2558	1.2605	0.7218
1				10.2994	0.2470	1.2826	0.7325
1.5				11.0638	0.2371	1.3037	0.7427
2.0				11.8132	0.2261	1.3237	0.7526
0.5	1.0			10.7047	0.2219	1.3061	0.7479
	1.5			11.8629	0.1771	1.3481	0.7728
	2.0			12.9970	0.1216	1.3873	0.7970
	0.5	1.0		13.6026	-0.1429	1.4305	0.8395
		1.5		15.3723	1.2633	1.4638	0.8029
		2.0		17.2365	1.2700	1.4630	0.8340
		0.5	0.5	9.1104	0.2799	1.3553	0.7818
			0.8	8.7405	0.2917	1.4499	0.8380
			1.0	8.5104	0.2941	1.5123	0.8738







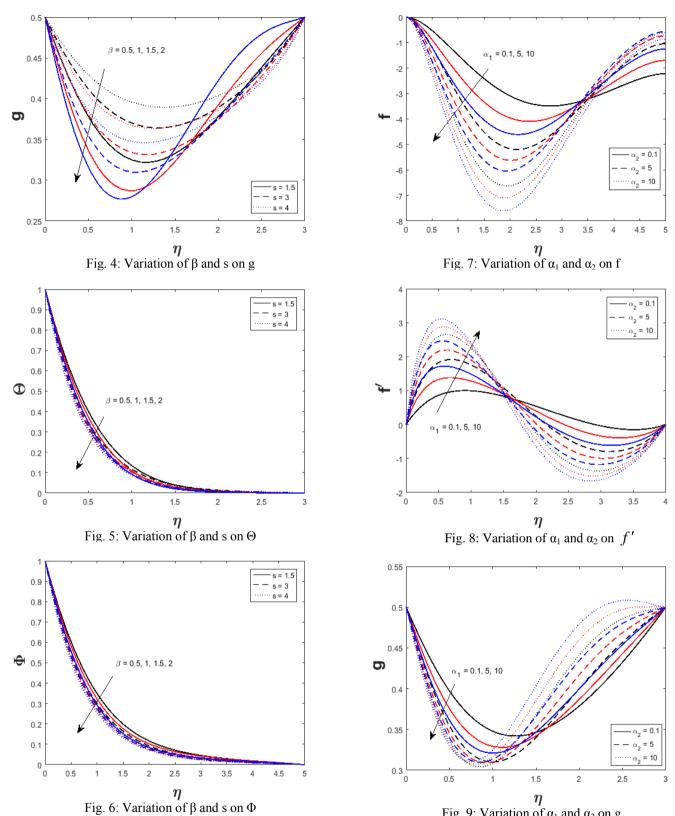


Fig. 9: Variation of α_1 and α_2 on g

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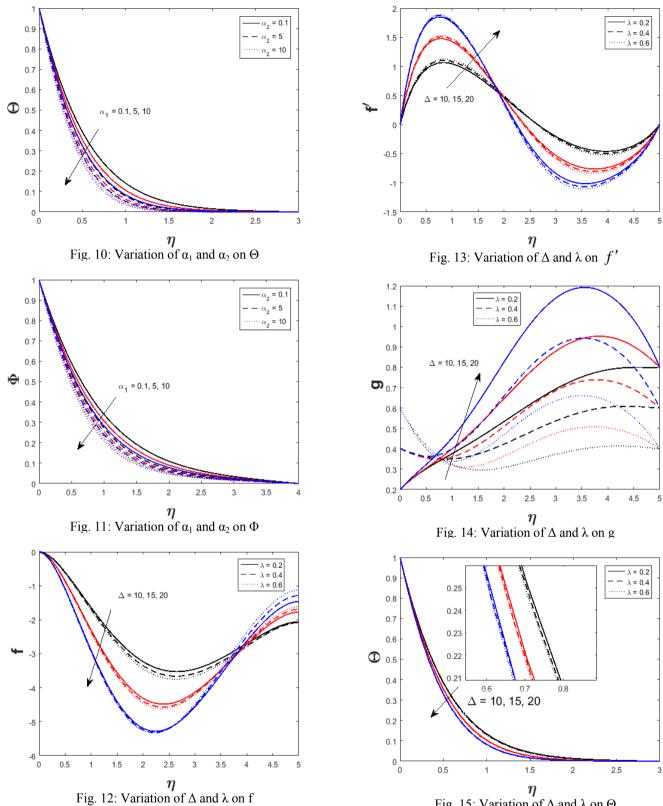
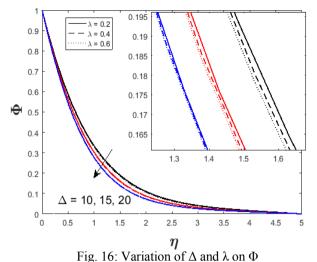


Fig. 15: Variation of Δ and λ on Θ

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V. CONCLUSIONS

This presents paper address the flow characteristics of non-Newtonian rotating fluid over a rotating cone in a Darcy porous medium. Resulting set of highly non-linear equations are solved using numerical technique. The various effect of fluid flow parameters on velocity, temperature concentration fields along with the Cfx, Cfy, Nu and Sh are presented with the help of graphs and tables.

The conclusions are as follows:

- Increase in unsteadiness parameter causes to accelerate tangential and azimuthal profiles while decelerates normal velocity, temperature and concentration fields.
- An increase in Casson fluid parameter enhances normal velocity filed while reduces tangential & azimuthal velocity, temperature and concentration fields.
- As non-linear thermal and concentration parameters increasing, tangential & azimuthal velocity, temperature and concentration fields are decreased while normal velocity increased.

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