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THE SURFACE STRESSES ON THE PROPAGATION OF ELASTIC WAVES

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***Abstract*:**The propagation of Rayleigh waves in non-homogeneous incompressible medium with an isotropic and homogeneous material boundary is studied. The period equation is obtained and it is compared with the corresponding equation of half-space with free boundary.
The frequency equation of Rayleigh waves in an incompressible non-local elastic medium under gravity effect is derived. This equation is solved numerically taking the particular form of non-local influence function as derived in lattice dynamics. The frequency is seen to decrease with an increase of gravity constant. The amount of decrease of frequency due to the increase of gravity, increases with the increase of wave number.
Rayleigh wave propagation in non-local elastic medium with material boundary is also studied. The important results of this study are: For a given material boundary the effect of the presence of non-locality is to increase the critical wave length. The velocity is an increasing function of Poisson's ratio. And for a given material the velocity corresponding to non-local case is less than classical case.

***Keywords*:**-Rayleish wave, surface stress, wave propagation, elastic waves, love waves, stoneley waves, transverse waves, elastic medium.

INTRODUCTION:

The study of deformations of elastic solids with stress supporting boundaries is of recent origin. A boundary that supports surface stresses is called a material boundary. The recent studies on wave propagation in elastic solids with material boundary have drawn the attention for divising experiments for the determination of surface elastic parameters and hence to aid the design of signal processing devices. In this chapter we study the influence of surface stresses on the propagation of elastic waves, in particular Rayleigh wave propagation in non-homogeneous bodies. Since the wave propagation in an elastic plate is analogous to propagation in layered spaces, we study the influence of surface stresses on elastic waves in an infinite plate.

Since velocity gradients are known to exist in the earth's curst and mantle, many solutions are not available as in the case of homogeneous medium. A number of workers, notable among them being Meissner [64], Jeffreys [48], Sezawa [92], Matuazawa [63] etc., have contributed to this theory. The striking feature of these studies is "the non-homogeneity increases the phase velocity of the longer period waves". All these studies were made taking the overlying layer of finite thickness.

The first discussion of Rayleigh wave propagation in non-homogeneous medium seems to have been given by Honda [44]. Sezawa [90] investigated general equations of wave propagation in a semi-infinite solid body of varying elasticity. Rayleigh waves in a semi-infinite incompressible medium where a layer in which rigidity varies linearly with depth is underlain by a uniform elstic substratum were investigated by Newlands [73], who also extended the investigations to compressible media. Stoneley [94] derived the period equation of Rayleigh waves in non-homigeneous incompressible medium.

In many respects, wave propagation in elastic plates is analogous to propagation in layered spaces. Oscillations of an elastic plate, the surfaces of which are free of stresses, were investigated by Rayleigh [84], Prescott [81] and Stato [87]. Lamb [61] has considered the propagation of plane waves in an infinite plate and analysed the dispersion equation for the lowest symmetric and anti symmetric modes.

Basic Equation:
S

1). Let the elastic body be the Cartesian region $x_3 > 0$ and the boundary plane $x_3 = 0$ is a material boundary surface. If the body is assumed to be homogeneous and isotropic, then, within the frame work of Linearized theory, the displacement $u_i(\vec{x},t)$ and stress $t_{ij}(\vec{x},t)$ $(\text{in } x_3 > 0)$ in the absence of body forces satisfy

$$t_{ij,j} = \rho \ddot{u}_i \tag{1}$$

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) \quad (i,j,k = 1,2,3) \tag{2}$$

where comma denotes differentiation with respect to x_j and δ_{ij} is kronecker delta. Further the surface stress tensor is given by

$$\Sigma_{\alpha\beta} = \delta_{\alpha\beta}\left[\sigma + (\lambda_0 + \sigma)u_{\gamma,\gamma}\right] + (\mu_0 - \sigma)\left(u_{\alpha,\beta} + u_{\beta,\alpha}\right) + \sigma u_{\alpha,\beta} \tag{3}$$

$$\Sigma_{3\beta} = \sigma u_{3,\beta} \tag{4}$$

in which α, β, γ take the values 1,2 only.

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If the boundary is free of external loads, the balance of linear momentum takes the form

$$\Sigma_{i\alpha,\alpha} + t_{i3} = \rho_o \ddot{u}_i \qquad \text{on } x_3 = 0 \tag{5}$$

2). Let two elastic bodies occupy respectively the regions $x_3 > 0$ and $x_3 < 0$ and that the plane $x_3 = 0$ be an elastic material interface. If the interface supports an interfacial stress given in terms of the tensor $\Sigma_{i\alpha}(x,t)$ then the balance of linear momentum takes the form

$$\Sigma_{i\alpha,\alpha} + t_{i3}^{(1)} - t_{i3}^{(2)} = \rho_0 \ddot{u}_i \text{ on } x_3 = 0 \tag{6}$$

where $t_{i3}^{(1)} (\text{in } x_3 > 0)$ and $t_{i3}^{(2)} (\text{in } x_3 < 0)$ are the stresses satisfying

$$t_{ij,j}^{(1)} = \rho_1 \ddot{u}_i^{(1)}, \quad \text{in } x_3 > 0$$

$$t_{ij,j}^{(2)} = \rho_2 \ddot{u}_i^{(2)}, \quad \text{in } x_3 < 0 \tag{7}$$

and the term $\sigma\delta_{\alpha\beta}$ (involved in $\Sigma_{i\alpha}$) is the residual interfacial stress. On the interface ($x_3 = 0$)

$$u_i^{(1)} = u_i^{(2)} = u_i$$

We assume the following restrictions on physical constants

$$\rho > 0, \lambda + 2\mu > 0, \mu > 0, \rho_0 > 0, \mu_0 > 0 \tag{8}$$

Rayleigh wave propagation in non-homogeneous elastic half-space with material boundary

Let a non-homogeneous isotropic elastic medium occupying the region $x_3 \geq 0$ be under an isotropic homogeneous elastic boundary surface ($x_3 = 0$). We assume a simple harmonic wave train traveling in the x_1 – direction such that:

(i) the disturbance is independent of the x_2 – coordinate.

(ii) It decreases rapidly with distance x_3 from the material surface.

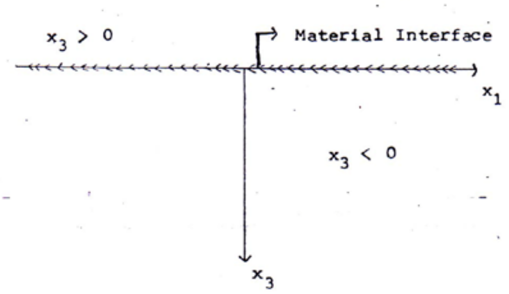

Fig 1: Geometry of the problem

Hence the displacement u_1 and u_3 are of the form

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}$$

$$u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1} \tag{9}$$

Where ϕ and ψ are potential functions satisfying the above condition which are functions of x_1, x_3 and t.

For two-dimensional motion in non-homogeneous elastic body the equation of motion (1) takes the form

$$\frac{\partial}{\partial x_1}\left[\lambda\Delta + 2\mu\frac{\partial u_1}{\partial x_1}\right] + \frac{\partial}{\partial x_3}\left[\mu\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right)\right] = P\ddot{u}$$

$$\frac{\partial}{\partial x_1}\left[\mu\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right)\right] + \frac{\partial}{\partial x_3}\left[\lambda\Delta + 2\mu\frac{\partial u_3}{\partial x_3}\right] = P\ddot{u} \tag{10}$$

where

$\Delta = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}$ is the cubical dilatation.

The non-vanishing components of surface stress of the material boundary (3) and (4) are

$$\sum\nolimits_{11} = \sigma + (\lambda_o + 2\mu_o) u_{1,1}$$

$$\sum\nolimits_{31} = \sigma u_{3,1} \tag{11}$$

And the equations of the balance of supporting stress (6) are

$$\sum\nolimits_{11,1} + t_{13} = \rho_0 \ddot{u}_1$$

$$\sum\nolimits_{31,1} + t_{33} = \rho_0 \ddot{u}_3, \qquad \text{on } x_3 = 0 \tag{12}$$

To make the mathematical calculation simple let the half-space be assumed to consist of incompressible solid for which $\lambda \to \infty$ as $\Delta \to 0$ in such a manner that lim $\lambda\Delta = -(\pi)$ remains finite.

Assuming the rigidity of the non-homogeneous half-space as $\mu = d_1 + d_2 x_3$. and using (12), the equations of motion (10) take the form

$$\frac{\nabla - \partial\Pi}{\partial x_1} + \mu\nabla^2 u_1 + d_2\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) - \rho\,\ddot{u}_1 = 0$$

$$\frac{\nabla - \partial\Pi}{\partial x_3} + \mu\nabla^2 u_3 + 2d_2\frac{\partial u_1}{\partial x_3} - \rho\,\ddot{u}_3 = 0 \tag{13}$$

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Differentiating the first of equations (13) with respect to x_1 and the second with respect to x_3 and then adding, we get

$$\pi = 2d_2 u_3 - \rho\ddot{\phi}. \tag{14}$$

Using this value of π, the equations (13) can be written as

$$\frac{\partial}{\partial x_3}\left[\mu\nabla^2\psi - \rho\ddot{\psi}\right] - \mu\frac{\partial}{\partial x_1}\left(\nabla^2\phi\right) = 0$$

$$\frac{\partial}{\partial x_1}\left[\mu\nabla^{-2}\psi - \rho\ddot{\psi}\right] + \mu\frac{\partial}{\partial x_3}\left(\nabla^2\phi\right) = 0 \tag{15}$$

For an incompressible material,

$$\Delta = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} = 0$$

i.e., $\nabla^2\phi = 0$ (16)

If we set

$$F = \mu\nabla^2\psi - \rho\ddot{\psi} = 0 \tag{17}$$

the equations (15) take the form

$$\frac{\partial F}{\partial x_1} = 0, \quad \frac{\partial F}{\partial x_3} = 0 \tag{18}$$

Thus, finally we have to solve the equation (16) and (17) to get ϕ and ψ.

If we assume the functions

$$\phi = \phi_o e^{ik(ct - x_1)}$$

$$\psi = \psi_o e^{ik(ct - x_1)} \tag{19}$$

the equations (16) and (17) become (on omitting the subscript 0)

$$\frac{d^2\phi}{dx_3^2} - k^2\phi = 0 \tag{20}$$

$$\frac{d^2\psi}{dx_3^2} - k^2\left(\frac{\rho c^2}{d_1 + d_2 x_3} - 1\right)\psi = 0 \tag{21}$$

Solving (20), we get

$$\phi = Ae^{-kx_3}\ \mathrm{Cos}\ K\left(ct - x_1\right) \tag{22}$$

The equation (21) can be transformed into

$$\frac{d^2\psi}{d\xi^2} + \left(\frac{q_1}{\xi} - \frac{1}{4}\right)\psi = 0 \tag{23}$$

Where

$$\xi = 2\left(kx_3 + q_2\right) - q_1 = \frac{q_2 c^2}{2_{q_o}},$$

$$q_2 = \frac{kd_1}{d_2}, \quad q_0 = \left(\frac{d_1}{c}\right)1/2.$$

The equation (23) is satisfied by Whittakar's function $w_{q_1},1/2\left(\xi\right)$. Therefore the solution of (23) is

$$\psi = BW_{q_1},1/2\left(2kx_3 + 2_{q_2}\right)\mathrm{Sink}\left(ct - x_1\right). \tag{24}$$

Substituting the values of ϕ and ψ in (9) we get the expressions for displacements. The equations of balance of supporting stresses (12) in terms of ϕ and ψ are (on x_3 = 0).

$$\left(\lambda_0 + 2\mu_0\right)\left(\frac{\partial^3\phi}{\partial x_1^3} + \frac{\partial^3\psi}{\partial x_3\partial x_1^2}\right) + d_1\left[\frac{2\partial^2\phi}{\partial x_1\partial x_3} - \frac{\partial^2\psi}{\partial x_3^2} + \frac{\partial}{\partial}\right.$$

$$-\rho_0\frac{\partial^2}{\partial t^2}\left(\frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}\right) = 0 \tag{25}$$

$$\sigma\left(\frac{\partial^3\phi}{\partial x_1^2\partial x_3} + \frac{\partial^3\psi}{\partial x_1^3}\right) - \Pi + 2d_1\left(\frac{\partial^2\phi}{\partial x_3^2} + \frac{\partial^2\psi}{\partial x_1\partial x_3}\right)$$

$$-\rho_0\frac{\partial^2}{\partial t^2}\left(\frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}\right) = 0 \tag{26}$$

On substituting ϕ and ψ from (22) and (24) the above equations give

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$$\left(k^2 + 2d_1 k - \rho_0 c^2 k^2\right)A + \left(-2\Gamma k^2 w' + 4kw'' + kd_1 w\right.$$

$$\left(\sigma k^2 + 2d_2 - \rho kc^2 + 2d_2 k - \rho_0 c^2 k^2\right)A + \left(\sigma k^2 w + 2\right.$$

Where $\Gamma = \lambda_0. + 2\mu_0.$

Eliminating the constants A and B from the above equations, we get the dispersion equation.

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$$\left(\frac{\Gamma^* k^2}{kd_1}+2\right)\left(\frac{\sigma^* k^2}{kd_1}+\frac{2}{q_2}-\frac{4w'(2q_2)}{w(2q_2)}\right)$$
$$=\left(2-\frac{c^2}{q_0^2}-\frac{2\Gamma^* k^2}{kd_1}\frac{W'(2q_2)}{W(2q_2)}\right)\left(\frac{\sigma^* k^2}{kd_1}+\frac{2}{q_2}+2-\frac{c^2}{q_0^2}\right) \quad (27)$$

Where

$$\Gamma^* = \Gamma - \rho_0 c^2$$
$$\sigma^* = \sigma - \rho_0 c^2.$$

If the boundary is of the conventional stress free type, we have $\sigma^* = 0 = \Gamma^*$. the equation (27) reduces to

$$\left(2-\frac{c^2}{q_0^2}\right)\left(\frac{2}{q_0^2}+2-\frac{c^2}{q_0^2}\right)=\frac{4}{q^2}-\frac{8w'(2q_2)}{w(2q_2)}$$

Conclusion:
This equation agrees with the period Equation and the waves continue to be Dispersive in this case also. Further, because of appearance of σ^*, Γ^* in the equation (27), the surface stress plays an important role in determining the characteristic quantities like velocity and wave length. The behaviour of the propagation may be attributed to the various possibilities appropriate to the values assumed by the parameters σ, Γ, ρ_0.

The period equation of Rayleigh waves in non-homogeneous incompressible medium under the effect of surface stress is derived and it is seen that this dispersion equation is in agreement with its classical counterpart in the absence of surface stress.

While studying the wave propagation in non-local elastic infinite plate, the period equation is derived and the long and short wave approximations are made to seek the nature of symmetric and Anti-symmetric oscillations. It is observed that these long and short waves depend on wave number whereas these waves in classical case are independent of wave number except the Anti-symmetric long waves.

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REFERENCE:-

1.	Eringen, A.C.	Continuum Physics, Vol. IV, Academic Press, N.Y. (1976)
2.	Eringen, A.C.	J. Phys. D. Appl. Phys., Vol.10, pp. 671-678 (1977)
3.	Honda. H.	Kisyo-syusi [9], Vol.6 p. 237 (1931).
4.	Jeffreys. H.	Monthly Notices Roy-Astron. Soc. Geophys. Suppl. Vol.2. 101-111 (1928).
5.	Lamb. H.	Proc. Roy. Soc. Lond. A.Vol. 93, pp. 114-128 (1916)
6.	Matuzawa	Bull. Earth quake research Inst. Vol. 6, 225-228 V. p. 213 (1929)
7.	Meissner. E.	Proc. Second. Inter. Congr. Appl. Mech., 3-11 (1926).
8.	Murdoch. A.I.	J. Mech. Phys. Solids., Vol. 24, pp. 137-146 (1976).
9.	Murdoch. A.I.	Q.J1. Mech. Appl. Math., 31, 531 (1978).
10.	Murdoch. A.I.	Int. J. Engng. Sci., 16. 131 (1978)
11.	Newlands. M.	Monthly Notice Roy. Astron. Soc. Geophys. Suppl. Vol. 6. pp. 109-125 (1950).
12.	Prescott. J.	Phil. Mag. Vol. 33. 703-754 (1942).
13.	Rayleigh. Lord.	Proc. Lond. Math. Soc. Vol. 20, p. 225 (1889).
14.	Sato. Y.	Bull. Earth quake, research Inst. Vol. 29, 223-261 (1951)
15.	Sezawa. K.	Bull. Earth quake research Inst. Vol. 9. pp.

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		310-315 (1931).
16.	Sezawa. K. and Kanaia. K.	Bull. Earth quake research Inst. Vol.17. 685-694 (1936).
17.	Stoneley. R.	Proc. Rat. Sci. A 106, 416-428 (1924).

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