



## A Lexi-Search Approach of Variant Vehicle Routing Problem

Dr. Sobhan Babu K

Associate Professor, Department of Mathematics, JNTUK  
University College of Engineering Vizianagaram,  
Vizianagaram, Andhra Pradesh, India  
[sobhanjntu@gmail.com](mailto:sobhanjntu@gmail.com)

Vijaya Kumar K

Assistant Professor, Department of Mathematics, Sri  
Vidyaniketan Degree College, Tirupati, Chittoor,  
Andhra Pradesh, India  
[vijayakumarh19@gmail.com](mailto:vijayakumarh19@gmail.com)

Purusotham S

Assistant Professor, Department of Mathematics,  
Statistics and Operational Research Division,  
VIT University, Vellore, Tamil Nadu, India  
[drpurusotham.or@gmail.com](mailto:drpurusotham.or@gmail.com)

**Abstract:** The present study addresses a variant of vehicle routing problem (VRP). Let there are  $n$ - nodes, among them a class of  $n_i$  ( $i < n$ ) nodes may act as source nodes for loading/reloading the goods and the rest of the nodes may require the goods which available at the source nodes. The requirement for the nodes is supplied through a vehicle, it has a finite capacity. The aim is to find an optimal trip schedule for the capacitated VRP which minimizes the total distance such that the vehicle has to traverse through a node exactly once according to the precedence relation and meet the requirements at the nodes. Here the precedence relation says that  $i < j$ , which means, before reaching the  $j^{\text{th}}$  node, an  $i^{\text{th}}$  node must covered in the trip schedule and it is not necessary that the  $j^{\text{th}}$  node is an immediate successor. The problem often can be modeled as a zero-one programming problem. The problem is having a variety of applications in logistics, networking theory, routing and allied areas. To solve the proposed problem we planned to develop an exact algorithm called Lexi-Search Algorithm. For implementation of the algorithm we would like to develop a suitable code using C-Language.

**Keywords:** Vehicle Routing Problem, Precedence Relation, Lexi-Search Algorithm, Zero-One Programming, Hamiltonian Cycle, Re-loading nodes.

### I. INTRODUCTION

The Problem of Routing of Vehicles, also known as: a VRP is an conjugable optimization and integral programming issue looking to serve an amount of the stations with the help of the vehicle [1]. The problem of VRP has been considered to be of great importance in several fields like logistics, networking theory, routing and allied areas along with transportation and distribution. Given in a general VRP is the set of nodes, including some source nodes. The general goal is to minimize the total cost/distance such that the requirement of all nodes gets satisfied and all nodes are to be traversed only once. Also, the starting source returns to its original position after traversing all the nodes as and when required, according to the algorithm and constraints. Several articles have been handy in order to have greater depths on the research as well as results [1]. The VRP is an empirical issue which has been foreviewed in a great extend in the OR literature [2]. VRP, a mentioned problem, is of great importance in the area of conjugable optimization issues, because of the two reasons, that it is of sensible application and a noteworthy level of hardness [3]. In order to have the optimum solution, we provide a lexi search method [4].

### II. LITERATURE SURVEY

Quondam researches betoken that Elucidation to Vehicle Routing Problem consists of the heuristics algorithms, the meta-heuristics algorithms and the precise algorithms [5]. Some other algorithms like Ant Colony

Algorithm, Genetic Algorithm, SAA, & TSA exists in order to obtain the solution of a bigger-scale Vehicle Routing Problem [5]. But still, unfortunately, there exists problems [5]. Till date, research along with a minimum spanning tree is still not yet revealed in order to get the solution of VRP. [5]. GLS, aka Guided Local Search is a recent meta-heuristic approach. But, the drawback is that it works by selecting a cost function with a penance period depending upon how close that search goes to earlier traverse. As a result, it inspires to have variegation. The drawback of the technique of local search is that the instability of the outcomes from one problem to another is large. Exact algorithms has been dependent upon the approach of branch and bound. The paper on problem of dispatching truck, that could be viewed as the higher level hierarchy of TSP has been formulated shown in [6]. Four more from China provide an allocation of the customers with multiple depots and a restriction on each [7]. A research on genetic issues and its applications, and, its hybrid version has been incorporated in [8] and [9] respectively.

The 2-level heuristic approach with the constraints [10] has been mentioned. The Lexi search approach in order to solve for TSP has been nicely implemented in [11]. An approximation approach with constraints of distance and capacity as a balanced has been mentioned in [12]. The general idea of an approximate along with exact methodologies has been briefly explained in [13]. The TSA along with its conjugation with programming with constraints has been given in [14]. An approach of metaheuristics with constraints has been mentioned in [15], and its combination with an exact approach is in [16]. The Lexi approach under a steerment of data in asymmetric TSP

has been incorporated in [17]. The contemporary algos. of metaheuristic approach for a problem of assignment has been incorporated in [18]. The study of a dynamic VRP wrt its esteem, along with contemplation has been incorporated in [19].

The consideration indicates that the proposed approach is determinative, breviloquent, convincing and reduces complexity of an algorithm in a notably manner.

### III. PROBLEM DEFINITION:

The present study addresses a variant of vehicle routing problem (VRP). Let there be  $n$ - nodes, among them a class of  $n_i$  ( $\ll n$ ) nodes may act as source nodes for loading/reloading the goods and the rest of the nodes may require the goods which are available at the source nodes. The requirement for the nodes is supplied through a vehicle, which consist of finite capacity. The aim is to find an optimal trip schedule for the capacitated VRP which minimizes the total distance such that the vehicle has to traverse through a node exactly once according to the precedence relation and meet the requirements at the nodes. Here the precedence relation says that  $i < j$ , which means, before reaching the  $j^{\text{th}}$  node, an  $i^{\text{th}}$  node must covered in the trip schedule and it is not necessary that the  $j^{\text{th}}$  node is an immediate successor. The problem often can be modeled as a zero-one programming problem.

The problem is having a variety of applications in logistics, networking theory, routing and allied areas. To solve the proposed problem we planned to develop an exact algorithm called Lexi-Search Algorithm. In other words, given are the set of nodes including some sources and a finite vehicle capacity. Our main objective is that we will minimize the total distance/cost of the total trip schedule such that requirement of each nodes is met at a time, and not partially, and such that each node is visited exactly once by a source node, i.e. a Hamiltonian cycle is to be formed. Let there be 'n' cities (nodes). We classify them into 2 parts such that 1 part contains the nodes that acts as source/sub-source in a way that they help in reloading the requirement in the source so that the source gets the requirement and can traverse efficiently to the other nodes forming a Hamiltonian cycle, and other group are the normal/ordinary nodes acting as a destination nodes. We require the sub-sources because there might exists a case where demand of the "to be traversed node" is higher than the capacity of the vehicle at a specified time. We provide the precedences to all the nodes.

The precedence constraint here is that we can visit the particular node, with higher precedence, only if the other predetermined node, with lower precedence is visited. It is NEITHER necessary that, the node, with lower precedence, before visiting a particular node, with a higher precedence, has to be visited first or in any order, but instead only like that it should be visited before the other higher precedence node, NOR that the other higher precedence node should come as just successor of the lower mentioned node, but instead only like that it should come after that mentioned lower precedence node. Note that the total constraints would be very less, which is required to be satisfied, Also, all the precedence constraints would have been given initially. In this paper, the lexicographic search algorithm has been implemented to solve a Vehicle Routing Problem (VRP) with precedence constraints. Lexicographic search

algorithms are useful and are also advantageous in many cases. The requirement of every nodes should be contended by a single vehicle. A Superlative routing by the vehicle having the finite capacity traversing the set consisting of the nodes having known demands can be considered as the capacitated routing issues of a vehicle, (CVRP). For implementation of the algorithm, we would like to develop a suitable code using C-Language.

### IV. NOTATIONS

The following are the considerations:-

The Nodal points:  $N = \{a_1, a_2, a_3, \dots, a_n\}$ .

Here,  $n$  = total number of all nodes..

The Set of vertices  $S = \{a_1, a_2, a_3, \dots, a_k\} \subseteq N$ , that acts as source nodes for loading/reloading the goods.

Set  $S^c = (N - S)$ , consists of the remaining nodes, that require the goods, acts as destination nodes.

i.e. The number of nodes in  $S^c$  = number of nodes in  $(N - S)$ , say  $m$ . Hence, we get that: -  $k + m = N$ .

A Distance Matrix,  $d_{ij}$  ( $n \times n$ ), which is a matrix in which the distance between each pair of nodes has been mentioned.

Note that the given matrix need not always need to be symmetric. Hence, it can be either symmetric or asymmetric. It is symmetric iff  $c_{ij} = c_{ji}$  for each  $i, j$  belonging to  $N$ . It also pleases an inequality of triangle iff  $c_{ij} \leq c_{ik} + c_{kj}$ , for each of  $i, j$  and  $k$  belongs to  $N$ .

The finite Requirement of all nodes of the system has been mentioned in an array of requirement, denoted by  $RQ[i]$ , where  $i$  ranges from the 1<sup>st</sup> node till all the nodes has been covered. i.e.  $RQ[i]$  denotes a requirement corresponding to the  $i^{\text{th}}$  station.

The Capacity of a Vehicle, denoted by  $V_k$ , is a finite quantity, which is not necessary to be  $\geq$  The summation of all  $RQ[i]$ , which is described in the above point.

The node '1' has been the initial and a source for the each possible routes of vehicle.

### V. MATHEMATICAL MODEL:

The given problem could be described as:-

$$\text{Minimize } z = \sum_{i=1}^N \sum_{j=1}^N X_{ij} d_{ij}$$

Subject to the following constraints:-

$$\sum_{i=1}^N x_{ij} \leq 1, \forall j \quad (\text{i})$$

$$\sum_{j=1}^N x_{ij} \leq 1, \forall i \quad (\text{ii})$$

$$\sum_i x_{ij} = 1, j \in S^c \quad (\text{iii})$$

$$\sum_j x_{ij} = 1, i \in S^c \quad (\text{iv})$$

$$\sum_i \sum_j x_{ij} \geq m \quad (\text{v})$$

$$X_{ij} = 1/0; \quad (\text{vi})$$

$$k \cdot V_k \geq \sum_{i=1}^N RQ[i] \quad (\text{vii})$$

$$|S| \cdot V_k = k \cdot V_k \quad (\text{viii})$$

Equation (i) indicates that summation of each individual rows in the matrix  $x_{ij}$  will be atmost 1.

Equation (ii) designates that summation of all individual columns of a matrix  $x_{ij}$  will be less than or equal to 1.

Equation (iii) and (iv) mentions that all the elements in  $S^c$  that are destination nodes has to be visited exactly once.

Equation (v) specifies that the comprehensive allocations would be greater than or equal to the number of the destination nodes.

An Equation (vi) stipulates that  $x_{ij}$  will be either 1 or 0. It would be 1, iff the requirement of the node 'j' gets supplied by a source 'i'. Else,  $x_{ij}$  will be equal to 0.

The next Equation (vii) implies that the product of the number of source nodes with the vehicle capacity would be greater than or equal to the summation of requirements of all the destination nodes.

Equation (viii) has been declared to get the feasible solution.

## VI. APPROACH

The Lexi Search Approach has been incorporated here in order to achieve a solution for a given problem of VRP with the precedence constraints. A sub-tour that has not been connected from a source, or a scenario where the overall aggregate weight becomes greater than capacity of a vehicle, are implicitly handled by this algorithm. In different words, creations of an invalid sub-tour are implicitly handled by this algorithm. It could also be mentioned that all the infeasible tours has not been allowed by the prescribed approach.

## VII. ALGORITHM

Step 1:- Construct the distance matrix  $d_{ij}(n \times n)$ , which is a matrix in which the distance between each pair of nodes has been mentioned. The inputs as a distance can be assigned in a random fashion that could be invoked by random [] function. Note that the given matrix need not always need to be symmetric. Hence; it can be either symmetric or asymmetric. It is symmetric iff  $c_{ij}=c_{ji}$  for each  $i,j$  belonging to  $N$ . It also pleases an inequality of triangle iff  $c_{ij} \leq c_{ik}+c_{kj}$ , for each of  $i,j$  and  $k$  belongs to  $N$ .

Step 2:- Perform Row-Reduction on the matrix  $d_{ij}(n \times n)$ , and correspondingly obtain the new matrix. Row-Reduction is incorporated as:- Find the minimum value row-wise individually, i.e. find the minimum value from each individual rows. Subtract all the corresponding elements of  $d_{ij}$  of all rows by the corresponding obtained minimum value for each rows. A Row-Reduction has thus been achieved.

Step 3:- From the obtained matrix in step-2, Perform Column-Reduction, and correspondingly obtain the new matrix again. It could be achieved as:- Find the minimum value column-wise individually, i.e find the minimum value from each individual columns. Subtract all the corresponding elements of all columns by the corresponding obtained minimum value for each column. A Column-Reduction has thus been performed.

Step 4:- Construct an Alphabet table from the matrix obtained in step-3. This is acquired as follows: From the 1<sup>st</sup> row, observe the minimum value, and its corresponding column number. Write the combination of them as the 1<sup>st</sup> element of an alphabet table. Repeat the same step for the next row and put in the alphabet table at the position of intersection of 1<sup>st</sup> row with 2<sup>nd</sup> column. Continue the same

procedure until each rows individually has been covered. Simultaneously, we get all the elements of 1<sup>st</sup> row of alphabet table. Now, choose a further greater integer and its associated column position from the 1<sup>st</sup> row of the matrix obtained in step-3. Write the combination of them as the 1<sup>st</sup> element of the 2<sup>nd</sup> row (next row) of an alphabet table. Repeat the same step for the next row and put in the alphabet table at the position of intersection of 2<sup>nd</sup> row with 2<sup>nd</sup> column. Continue the same procedure until each rows individually has been covered. Concurrently, we get all the elements of 2<sup>nd</sup> row of alphabet table.

Prolong the above procedure until all the rows of an alphabet matrix has been filled.

Step 5 : Construction of lexi search table.

Step 6: Find optimal path.

## VIII. NUMERICAL ILLUSTRATION:

We take a 8 x 8 matrix which is considered as a distance matrix  $d_{ij}$ , where each value of a matrix represents the distance between that value's corresponding  $i^{\text{th}}$  row and  $j^{\text{th}}$  column.

The nodal points are :  $N=\{1,2,3,4,5,6,7,8\}$ .

SubSource nodes,  $S=\{1,6\}$

Destination nodes requiring goods,  $S^c =\{2,3,4,5,7,8\}$ .

Requirement,  $RQ[i]=\{15,20,30,25,40,35\}$ , where  $i \in S^c$ .

Vehicle Capacity,  $V_k = 100$ .

The precedence relations are:-  $5 < 7$  and  $2 < 8$ .

Following is the table generated randomly by the computer.

Here, it is noteworthy to consider that an alphabet 'A' mentioned in any space represents the maximum possible number.

Table: 1

1	2	3	4	5	6	7	8	
A	2	5	6	0	7	4	10	0
3	A	14	18	5	3	2	13	2
11	0	A	3	16	13	1	16	0
19	4	8	A	19	16	18	16	4
9	12	13	6	A	10	14	2	2
8	2	13	9	9	A	11	6	2
17	5	3	8	13	3	A	10	3
12	12	0	15	5	9	7	A	0

The last column of a above matrix represents the minimum of the corresponding rows. Here,  $\alpha_1=0, \alpha_2=2, \alpha_3=0, \alpha_4=4, \alpha_5=2, \alpha_6=2, \alpha_7=3, \alpha_8=0$  are the minimum value respectively from each rows. Also, it can be

seen that :-  $\sum_{i=1}^8 \alpha[i] = 13$ .

After performing a Row-Reduction, the following matrix has been obtained.

Table: 2

1	2	3	4	5	6	7	8
A	2	5	6	0	7	4	10
1	A	12	16	3	1	0	11
11	0	A	3	16	13	1	16
15	0	4	A	15	12	14	12
7	10	11	4	A	8	12	0
6	0	11	7	7	A	9	4
3	2	0	5	10	0	A	7
12	12	0	15	5	9	7	A
1	0	0	3	0	0	0	0

Here, it has been observed that the last row includes  $\beta_1=1, \beta_2=0, \beta_3=0, \beta_4=3, \beta_5=0, \beta_6=0, \beta_7=0, \beta_8=0$ , which are minimum value respectively from each column from the above matrix. Also,  $\sum_{i=1}^8 \beta[i] = 4$ .

After performing Column Reduction, the following matrix has been obtained.

Table: 3

1	2	3	4	5	6	7	8
A	2	5	3	0	7	4	10
0	A	12	13	3	1	0	11
10	0	A	0	16	13	1	16
14	0	4	A	15	12	14	12
6	10	11	1	A	8	12	0
5	0	11	4	7	A	9	4
2	2	0	2	10	0	A	7
11	12	0	12	5	9	7	A

It can be noted that the summation of all  $\alpha$ 's and  $\beta$ 's = 17 (13+ 4).

Now, the alphabet table has to be constructed from the above matrix.

**IX. ALPHABET TABLE**

Table: 4

1	2	3	4	5	6	7	8
0-5	0-1	0-2	0-2	0-8	0-2	0-3	0-3
2-2	0-7	0-4	4-3	1-4	4-4	0-6	5-5
3-4	1-6	1-7	12-6	6-1	4-8	2-1	7-7
4-7	3-5	10-1	12-8	8-6	5-1	2-2	9-6
5-3	11-8	13-6	14-1	10-2	7-5	2-4	11-1
7-6	12-3	16-5	14-7	11-3	9-7	7-8	12-2
10-8	13-4	16-8	15-5	12-7	11-3	10-5	12-4
A-1	A-2	A-3	A-4	A-5	A-6	A-7	A-8

Following is the construction of lexi search table:-

It is to be noted that the letter 'R' in a following table indicates "REJECTED".

Table: 5

1	2	3	4	5	6	7	8
1-5 (0)	5-8 (0) R						
	5-4 (1)	4-2 (1)	2-1 (1) R				
			2-7 (1) R				
			2-6 (2)	6-2 (2) R			
				6-4 (6) R			
				6-8 (6)	8-3 (6)	3-2 (6) R	
						3-4 (6) R	
						3-7 (7)	7-3 (7) R
							7-6 (7)

							R
						R	7-1 (9) R
					R	3-1 (16)≥LB R	
				R	8-5 (11) ≥LB R		
				6-1 (7) R			
			R	6-5 (9) ≥LB R			
			2-5 (4) R				
		R	2-8 (12) ≥LB R				
		4-3 (5)	3-2 (5)	2-1 (5) R			
				2-7 (2) R			
				2-6 (6)	6-2 (6) R		
				R	6-4 (10) ≥LB R		
					2-5 (8) R		
				R	2-8 (16) ≥LB R		
				3-4 (5) R			
				3-7 (6) R			
			R	3-1 (15) ≥LB R			
		R	4-6 (13) ≥LB R				
		5-1 (6) R					
		5-6 (8) R					
R		5-2 (10) ≥LB R					
1-2 (2)	2-1 (2) R						
	2-7 (2)						
	R						
	2-6						

	(1) R						
	2-5 (5) R	5-8 (5) R	8-3 (5) R				
		R	8-5 (10) ≥LB R				
		5-4 (6) R	4-2 (6) R				
		R	4-3 (10) ≥LB R				
	R	5-1 (11) ≥LB R					
R	2-8 (13) ≥LB R						
1-4 (3)	4-2 (3)	2-1 (3) R					
		2-7 (3) R					
		2-6 (4) R					
		2-5 (6)	5-8 (6) R				
			5-4 (7) R				
		R	5-1 (12) ≥LB R				
	R	2-8 (14) ≥LB R					
	4-3 (7)	3-2 (7)	2-1 (7) R				
			2-7 (7) R				
			2-6 (8)	6-2 (8) R			
			R	6-4 (12) ≥LB R			
		R	2-5 (10) ≥LB R				
		3-4 (7) R					
		3-7 (8) R					
	R	3-1 (17) ≥LB R					
R	4-6 (15) ≥LB						

	R						
1-7 (4) R							
1-3 (5)	3-2 (5)	2-1 (5) R					
		2-7 (5) R					
		2-6 (6) R					
		2-5 (8)	5-8 (8)	8-3 (8) R			
			R	8-5 (13) ≥LB R			
		R	5-4 (9) ≥LB R				
	R	2-8 (16) ≥LB R					
	3-4 (5)	4-2 (5)	2-1 (5) R				
			2-7 (5) R				
			2-6 (6)	6-2 (6) R			
			R	6-4 (10) ≥LB R			
			2-5 (8)	5-8 (8) R			
			R	5-4 (9) ≥LB R			
		R	2-8 (16) ≥LB R				
	R	4-3 (9) ≥LB R					
	3-7 (6) R						
R	3-1 (15) ≥LB R						
1-6 (7) R							
1-8 (10) ≥LB R							
1-1 A≥LB R							

Here, in the construction of above lexi search table, the value '9' has been obtained as the lower bound, which is a feasible solution. Checking then, for all further possible optimal solutions, it has been observed that their values has increased than '9'. So, it has been concluded here that in an given example, the optimal solution is the path 1-5-4-2-6-8-3-7-1, with the total cost/distance appearing out to be '9'.

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