



## Population Projection Using Geometric Growth Model

Adeoye. Akeem. O.\*

Department Of Mathematics/Statistics  
Kwara state polytechnic, Ilorin, Nigeria  
akeemwaju@yahoo.com

Babalola J. B.

Department Of Mathematics/Statistics  
federal polytechnic, offa Nigeria  
babalolabam idele@gmail.com

**Abstract:** This research, population projection using Geometric Growth model, aimed to determine a particular time a population will double its size and the particular time it will be half of its size. We discover that population changes as time progresses which depends directly on the rate of increase  $\lambda$  in turn depend on the per capital rate of birth and death through their differences only. When we equate the time the equation will double it self to the time the population will be half of its size, we discover that the  $\lambda = \pm 2$ , which indicate  $t = \pm 1$  which implied an increases in population at  $\lambda = 2$  and a decrease in population at  $\lambda = -2$ .

**Keywords:** Population, projection, Geometric model, population changes, and rate of increase

### I. INTRODUCTION

The term "population growth" refers to how the number of individuals in a population increases (or decreases) with time. This growth is controlled by the rate at which new individuals are added to the population -- the birth rate, and the rate at which individuals leave the population -- the death rate[1].

There are many types of plants and animals, and different types show different kinds of population growth[2]. In this research we will consider a very simple type of growth where the animals in one generation give birth to the next generation. One of the greatest dangers planet Earth faces today is that of over-population by humans. Since the beginning of human history our population has been growing exponentially[3] (actually a little faster than exponentially). However, there is now little additional land that can be put into use for food production and our wastes are rapidly polluting the environment. Unless major technological breakthroughs are made, we are close to the the Earth's carrying capacity. Compounding the problem is the fact that our average birth rate is still very high It is possible we will overshoot the Earth's carrying capacity and for the first time in recorded history have massive population decline brought on by disease and starvation[4]. You have seen how sensitive a population's growth is to the birth rate. The decrease in birth rate needed to avert a disaster is not large, but it is one that everyone needs to work on together to achieve. The potential (or lack) for a population to increase Determine the consequences of changes in the current demography[4].

### II. METHODOLOGY

#### A. Population Models in General:

Observables,  $N$  or  $N(\text{age})$  or  $N(\text{stage})$  Project population size  $N$  as a function of time  $t$  Projection in terms of fundamental parameters describing demographic events in an individual's life e.g.,  $\text{Pr}(\text{birth})$ ,  $\text{Pr}(\text{death})$  enable understanding of how demographic vital rates affect the whole population No population experiences unlimited resources Yet, all populations have potential for exponential growth Projections[4].

#### B. Geometric Growth Models:

General motivation Sequence of population sizes through time  $N_t, N_{t+1}, N_{t+2}, \dots$  Change from one time to next ,increases due to births during period ,decreases due to deaths during period ,increases due to immigrants during period ,decreases due to emigrants during period[5].

### III. MATHEMATICAL FORMULATION

Population size after an interval of time

$$N_{t+1} = N_t + B + -D + I - E \dots (1)$$

$B$ =birth,  $D$ = death  $I$  = immigration,  $E$ = emigration

Change in population size

$$\begin{aligned} \Delta N &= N_{t+1} - N_t \\ &= B - D + I - E \dots (ii) \end{aligned}$$

### IV. ASSUMPTION OF GEOMETRIC MODEL

In a Closed population  $I = E = 0$ , Constant per capital birth ( $b$ ) and death ( $d$ )

$B = bt$ ,  $D = Dt$  ,  $b$  and  $d$  identical for all individuals regardless of genotype

No age- or size-structure,  $b$  and  $d$  identical for all individuals regardless of size, age. No time lags and birth and death depend on current population only.

Projection of Population Size

$$\begin{aligned} N_{t+1} &= N_t + B - D \\ &= N_t + \left( \frac{B}{N_t} - \frac{D}{N_t} \right) \cdot N_t \\ &= N_t + (b - d) \cdot N_t \\ &= (1 + (b - d)) \cdot N_t \\ &= (1 + R_t) \cdot N_t \\ &= \lambda_t \cdot N_t \\ \lambda_t &= \frac{N_{t+1}}{N_t} \end{aligned}$$

change in population size

$$\begin{aligned}
\Delta N &= N_{t+1} - N_t \\
&= (1 + R_t) \cdot N_t - N_t \\
&= N_t + R_t N_t - N_t \\
&= R_t N_t \\
R_t &= \frac{\Delta N}{N_t}
\end{aligned}$$

Finite rate of increase= $\lambda$

$$N_{t+1} = \lambda N_t$$

$$\lambda_t = \frac{N_{t+1}}{N_t}$$

Population will increase when  $\lambda > 1$

Population will decrease when  $\lambda < 1$

Population stable when  $\lambda = 1$

Assume a constant value of  $\lambda$  that is  $\lambda_t = \lambda$

$$N_t = \lambda N_{t-1}$$

$$= \lambda(\lambda N_{t-2})$$

$$= \lambda(\lambda(\lambda N_{t-3}))$$

$$= \lambda^t N_0$$

To determine how long it will take the population to double its size, that is, how long it will take the population to change from  $N_0$  to  $2N_0$

$$N_t = \lambda_t N_0$$

$$2N_0 = \lambda^t N_0 \dots (iii)$$

$$\lambda^t = 2$$

$$\ln(\lambda^t) = \ln(2)$$

$$t = \frac{\ln 2}{\ln \lambda}$$

To determine how long it will take the population to become half as large as its size, that is, how long it will take

the population to change from  $N_0$  to

$$N_t = \lambda^t N_0$$

$$\frac{1}{2} N_0 = \lambda^t N_0 \dots (iv)$$

$$\frac{1}{2} N_0 \lambda^t = \frac{1}{2}$$

$$\ln(\lambda^t) = \ln\left(\frac{1}{2}\right)$$

$$t = -\frac{\ln(2)}{\ln(\lambda)}$$

## V. CONCLUSION

It shows that a population changes size as time progresses and this depends directly on the finite rate of increase, in turn depends on the per capita rates of birth and death (through their difference only) to measure the rate of increase and to measure the potential for a population to grow.

## VI. REFERENCES

- [1]. Foll, M. and O. Gaggiotti "Identifying the Environmental Factors That Determine the Genetic Structure of Populations." *Genetics* **174**(2): 875-891. (2006).
- [2]. Francois, O., M. Currat, et al. "Principal component analysis under population genetic models of range expansion and admixture." *Mol Biol Evol* **27**(6): 1257-68. (2010)
- [3]. Hamilton, G., M. Currat, et al.. "Bayesian estimation of recent migration rates after a spatial expansion." *Genetics* **170** (1): 409-17. (2005)
- [4]. Hewitt, G. M. "Genetic consequences of climatic oscillations in the Quaternary." *Philos Trans R Soc Lond B Biol Sci* **359** (1442): 183-95; discussion 195. (2004)
- [5]. Bittinger, M. L., and Davic. *Intermediate Algebra: Concepts and Applications*. 6th ed. Reading, MA: Addison-Wesley Publishing. (2001)